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SOME REMARKS ON DYNAMIC SOARING

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SOME REMARKS ON DYNAMIC SOARING

Dr. E. Fritsch[1]

ABSTRACT. Extensive idealization of dynamic soaring makes it possible to treat the problem completely analytically. The influence of wind shear, air speed, lift-drag ratio and the angle of inclination of the flight path on dE/dh is calculated. A relationship is also derived for the anticipated cruising speed in the jet stream. The most satisfactory air speed and optimum angle of inclination of the flight path are determined for the given values. The cruising speeds and gains of energy to be expected in the jet stream are estimated numerically. Some hints for the pilot regarding flight in the jet stream are given in the paper.

Some Remarks on Dynamic Soaring

The glider pilot uses available thermals as well as upwinds and ascending /1* air currents as sources of energy. The albatross shows us that it is technically possible to make longer soaring flights by certain flight maneuvers in a layer where the wind intensity varies with altitude. The wind shear is particularly strong in the jet stream. Our knowledge in this area has recently been expanded [1], and has reached the point where we can expect the first tests using gliders to begin in the near future [2].

The principle of dynamic soaring flight can be explained by a numerical example.

The wind velocity at a given altitude is 10 km/h greater than the wind velocity at the surface of the water. An albatross is flying against the wind a short distance above the water, at 50 km/h (Figure 1, 1). When it climbs to to layer having a velocity 10 km/h greater, it flies at a speed of 60 km/h relative to the surrounding air. It then turns so that it is flying with the wind at a speed of 60 km/h relative to the surrounding air (3). Its speed relative to the air is thereby retained. This means that it is traveling 70 km/h relative to the ground. It retains this speed relative to the ground even

[1]Aeroclub of the GDR.
*Numbers in the margin indicate pagination in the foreign text.

while descending to zero altitude at (4). As we can see, the albatross gains considerable speed without physical exertion.

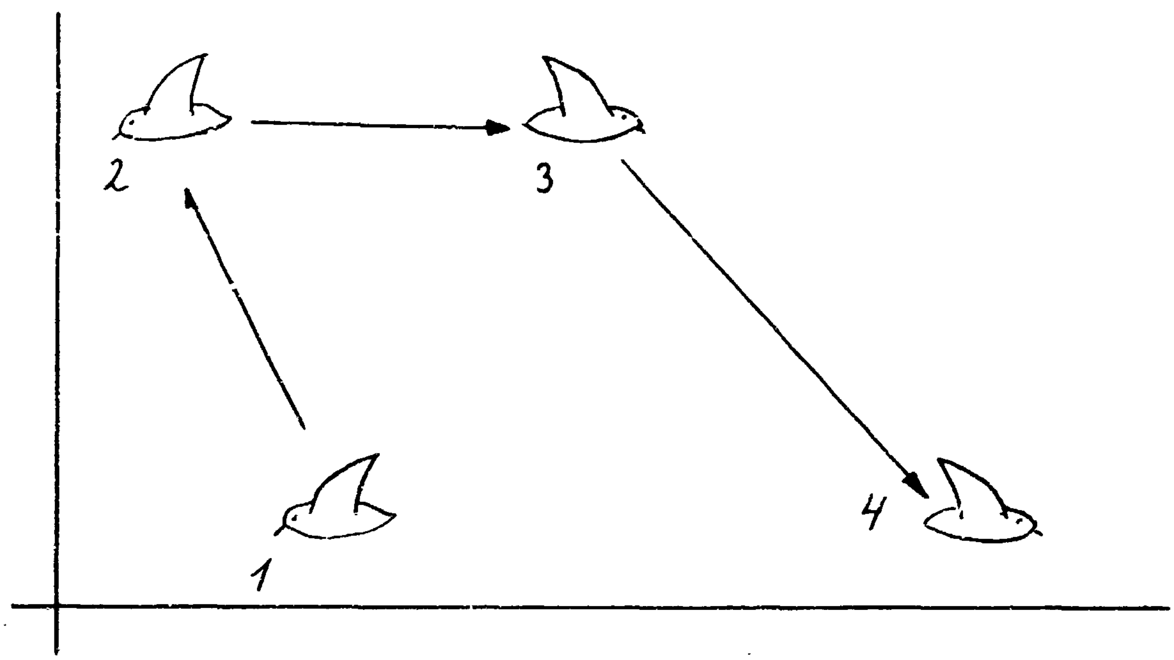


Figure 1.

The change in velocity owing to the conversion of potential energy and the wind velocity that may be present at zero altitude need not be taken into consideration in this cyclic process. Their effects cancel each other out in the long run.

Can we do the same as the albatross?

1. Energy Balance During Flight in a Current with Strong Wind Shear /1

The aircraft flies on a path with an instantaneous angle of inclination d (Figure 1). The equation of motion is as follows:

$$M \frac{dv^{+}}{dt} = -Mg \sin \qquad -W(v) \qquad (1)$$

V^{+} is the speed of an inertial system. This speed is measured by an outside observer.

V is the speed relative to the surrounding air. This speed is shown on an indicator in the aircraft.

M is the mass of the aircraft.

g is the acceleration due to gravity.

W (V) is the air resistance of the aircraft.

From Figure 2 we can see that

$$\vec{V}^{+} = \vec{V} + \vec{W}\ (h)$$

$\vec{w}$ (h) is the speed of the wind as a function of the altitude.

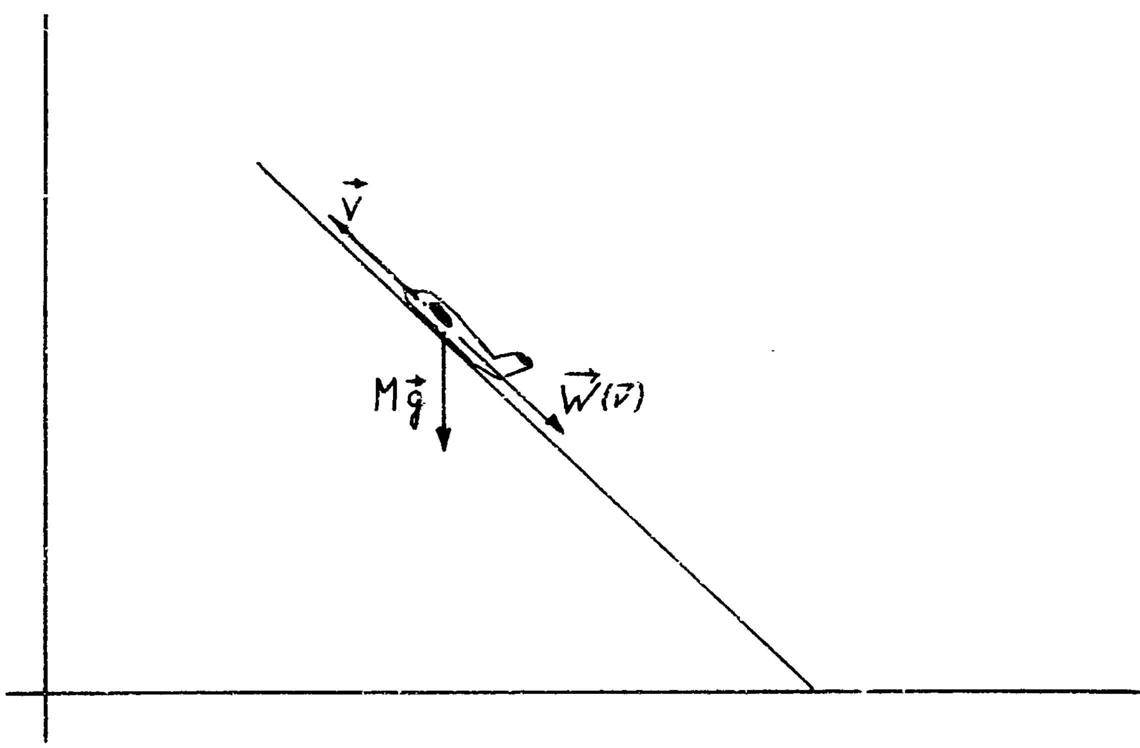


Figure 2.

Let the angle between $\vec{V}$ and $\vec{V}^{+}$ be small. We will then have as the components in the direction of motion:

$$V^{+} = V - W\ (h)\ \cos\alpha \tag{2}$$

If we consider

$$W\ (v) = Mg\varepsilon \tag{3}$$

(ε = lift-drag ratio)

and

$$h^{*} = V \sin\alpha \tag{4}$$

we will have from Equations (1) to (4)

$$M \frac{dv}{dt} = -Mg \cdot \sin \alpha - Mg \cdot \varepsilon - Mv \frac{dw(h)}{dh} \sin \alpha \cos \alpha \tag{5}$$

In the following, we shall use u to represent the change in wind velocity with altitude, $\frac{dw}{dh}$. /2

Is it possible to make the energy balance positive during one flight phase?

The energy balance is obtained in the usual way from the equation of motion (5) by multiplying it by the velocity and rearranging the equation. We then have

$$\frac{dE}{dt} = \frac{d}{dt} \left(\frac{M}{2} V^2 + Mgh\right) = Mv \left(vu^* \sin \alpha \cos \alpha - g\varepsilon\right) \tag{6}$$

The energy gain with height is more interesting for the problem at hand than is the energy gain with time.

With (4) and (6), this relationship will be

$$\frac{dE}{dh} = \frac{E^*}{h^*} = M \left(vu \cos \alpha \frac{g\varepsilon}{\sin \alpha}\right) \tag{7}$$

where $\frac{dE}{dh}$ depends on the angle of inclination of the flight path.

From $\frac{d}{d\alpha} \left(\frac{dE}{dh}\right) = 0$, we have the definitive equation

$$\sin \alpha = \sqrt[3]{\frac{g \cdot \varepsilon}{u \cdot v}} \cdot \sqrt[3]{\cos \alpha} \tag{8}$$

for the most favorable angle of inclination.

In most cases, $\sqrt[3]{\cos \alpha}$ can be equated to 1 (for example, $\sqrt[3]{\cos 40^\circ} = 0.91$). /3 A comparison of the solution by means of the approximation $\sqrt[3]{\cos \alpha} = 1$ with a graphic solution yielded an error of 1° for the values given in this section.

According to (3), the change in speed with altitude can amount to 20 knots/1000 feet (or $3.28 \cdot 10^{-2}\ S^{-1} = 3.28 \frac{m/s}{100\ m}$). The performance data of the glider give $1/\varepsilon = 33$ at V = 200 km/h. This information assumes that the air density 10 km up is only 25% of the value near the ground. All values given for normal air densities are therefore multiplied by the factor 2. The performance data of the "Foka 4A" glider were used in calculating all numerical values (Table 1).

TABLE 1 /15

V km/h	$1/\varepsilon$	$\varepsilon/V \cdot 10^{-3}$ $\frac{s}{m}$
160	32	0.7
186	34	0.57
200	33	0.54
240	27	0.55
280	22	0.59
320	17.3	0.65
360	14.3	0.7
400	12	0.75
440	10	0.82

The table is based on the performance figures for the Foka 4A glider. The speeds given by the manufacturer are multiplied by a factor of 2, in order to take into account the fact that the air density 10 km above sea level is four times less.

For $u = 3.28 \cdot 10^{-2}\ S^{-1}$ we obtain the values

$$\alpha = 33°$$

and

$$\frac{dE}{dh} = 0.1\ Mg$$

This corresponds to

$$\frac{dE}{dt} = 3\ Mg^{m/s}$$

or a climb of 3 m/s in the thermal. For the sake of demonstration, let $v \cdot u = 0.2\ g$.

The corresponding values for $u = 1.64 \cdot 10^{-2}\ S^{-1}$ are

$$\alpha = 39°$$

$$\frac{dE}{dh} = 0.025\ Mg$$

$$\frac{dE}{dt} = 0.8\ Mg\ m/s$$

How must the pilot fly in order to attain the calculated values? He knows /4 neither the most suitable angle of climb nor the value of u.

A pilot cannot calculate any extreme values in flight; for this reason, an instrument must take over the job. In flights in the regime under discussion, the pilot should have an instrument that shows him the value $\frac{dE}{dh}$. It can be calculated most simply by means of the equation

$$\frac{dE}{dh} = \frac{E^*}{h^*}$$

This relationship is exactly valid. The value $\frac{dE}{dt}$ is provided by the familiar "Tevar" and $\frac{dh}{dt}$ by the still more familiar rate-of-climb meter. The quotient is obtained from the two values. This is a task that the instrument industry can probably handle.

The pilot must control the angle of climb so that $\frac{dE}{dh}$ is constantly at a maximum. The most favorable angle will be somewhat less than 45°.

The above is a typical statement of the problem for soaring flight. During flight in a thermal, for example, the most favorable banking angle and speed in circular flight are known approximately. They must be re-established at each new level with considerable piloting skill. The sole criterion is that the climb must be as great as possible.

2. Estimating the Cruising Speed in the Jet Stream

The energy gain is greater than zero. This makes it possible to fly along a path.

The flight tactics are as follows:

To begin with, the pilot flies upward along a path with a large angle of inclination into a layer with a higher wind velocity (Figure 4) and then down again on a path with a smaller angle of inclination. In order to keep the calculations simple, we will assume that the difference in height is so slight that the changes in speed caused by flying upward and downward remain small relative to the air speed. We then can assume that the speed is constant during the flight.

The cruising speed V_R is obtained by definition from /5

$$V_R = \frac{X^* - X}{t^* + t} \tag{9}$$

(see Figure 4 for symbols).

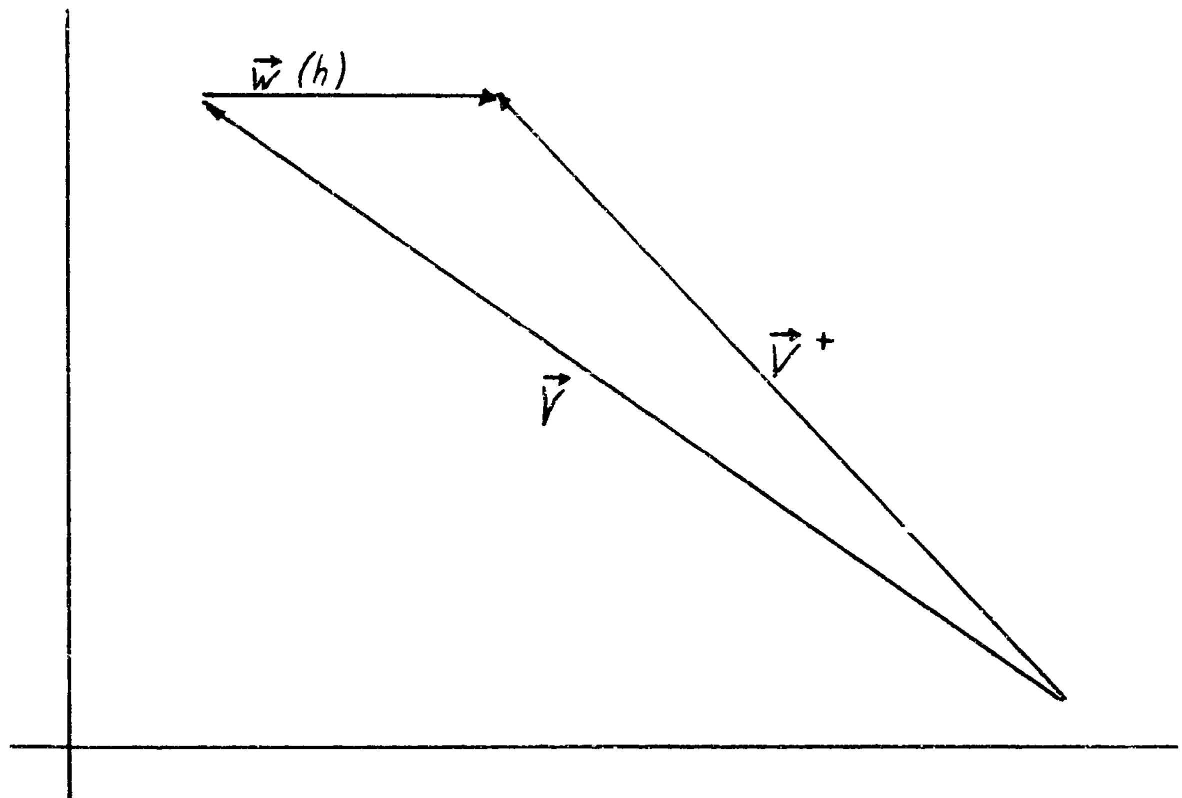


Figure 3.

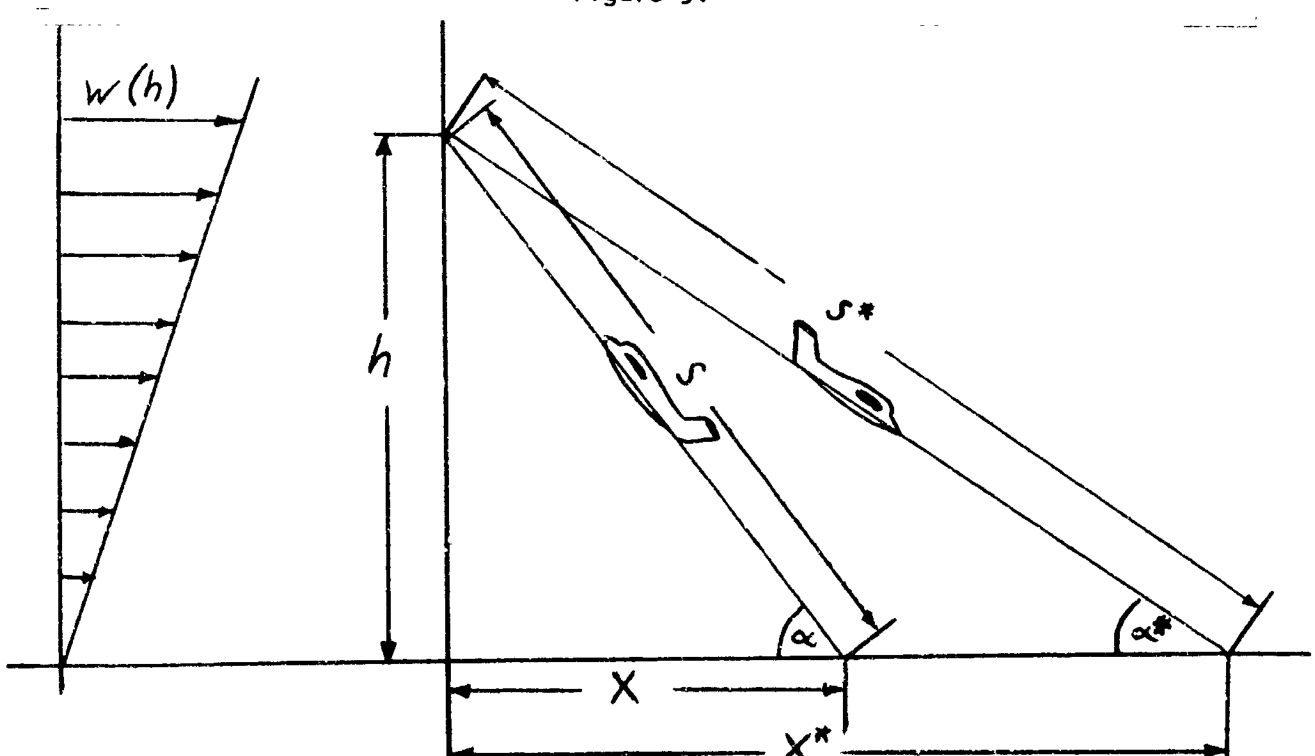


Figure 4.

Here t is the duration of the climb and t* is the duration of the descent.

With the aid of $t^* = S^*/v$, $t = s/v$ and elementary angle relationships from Figure 4, we obtain

$$V_e = V \frac{\sin(\alpha - \alpha^*)}{\sin \alpha + \sin \alpha^*} \qquad (10)$$

The angles α and α^* are not chosen freely. It is necessary to ensure that the cycle of climbing and descent can be repeated continuously. The integral of the energy balance during a cycle must be zero, i.e.,

$$\oint dE = \oint_{\text{cycle}} \frac{dE}{dh} \cdot dh = \int_{\text{climb}} \frac{dE}{dh} \cdot dh + \int_{\text{descent}} \frac{dE}{dh} \cdot dh = 0 \qquad (11)$$

Within the framework of our approximation, we obtain with (7) and $h_{\text{climb}} = h_{\text{descent}}$

$$\cos \alpha - \frac{g \cdot \varepsilon}{u \cdot v \cdot \sin \alpha} + \cos \alpha^* - \frac{g \cdot \varepsilon}{v \cdot u \cdot \sin \alpha^*} = 0 \qquad (12)$$

The calculation of the extreme value of $V_R = V_R(\alpha)$ with secondary condition (12) involves vague calculations. For this reason, we have calculated two versions, of which the more suitable will be selected for further study.

Version 1

The climb takes place along a path with maximum dE/dh. The descent is such that Equation (12) is fulfilled. This means the following flight tactics for the pilot: /6

The climb is made in the manner described in Section 1. The pilot notes the value of $\frac{dE}{dh}$. The angle of inclination is reduced until $\frac{dE}{dh}$ is negative and of the same magnitude as in the climb. This type of flight guarantees that the cycle can be repeated in full.

With an aircraft of the given performance class (Table 1), we will have for $u = 3.28 \cdot 10^{-2}\ S^{-1}$

$$\alpha^* = 7°$$

and

$$V_R = V \cdot 0.65 = 130 \text{ km/h}$$

The corresponding values for $u = 1.64 \cdot 10^{-2}\ S^{-1}$ are

$$\alpha^* = 16°$$
$$V_R = V \cdot 0.44 = 88\ \text{km/h}$$

Version 2

The pilot controls the angles of climb and descent so that $\frac{dE}{dh}$ is equal to zero. Equations (11) and (12) are thereby always fulfilled. Since $\frac{dE}{dt} = \frac{dE}{dh}\ V \sin \alpha$, the pilot requires for this flight regime only a "Tevar" in addition to his regular instruments.

For the same angle α, $\frac{dE}{dt}$ and $\frac{dE}{dh}$ have a value of zero. The function

$$\frac{dE}{dt} = M(v \cdot u \cdot \sin \alpha \cdot \cos \alpha - g\varepsilon) = M(\frac{u \cdot v}{2} \sin 2\alpha - g \cdot \varepsilon) = 0$$

has two zero points for $0 \leq \alpha \leq 90°$. They are calculated from

$$\sin 2\alpha = 2\ \frac{g \cdot \varepsilon}{v \cdot u} \tag{13}$$

For $VU \gg g \cdot \varepsilon$ they are approximately

$$\cos \alpha \approx \frac{g \cdot \varepsilon}{v \cdot u} \qquad \sin \alpha \approx 1$$
$$\sin \alpha^* \approx \frac{g \cdot \varepsilon}{v \cdot u} \qquad \cos \alpha^* \approx 1 \tag{14}$$

The error in approximation in this section has a maximum of 6%.

The calculation of the cruising speed is considerably simplified by (14). /7
We will have

$$V_R = V\ (1 - \frac{g\varepsilon}{v \cdot u}) \tag{15}$$

For $u = 3.28 \cdot 10^{-2}\ S^{-1}$ we will have a cruising speed of

$$V_R = V \cdot 0.84 = 168\ \text{km/h}$$

The angles are:

$$\alpha = 80°$$
$$\alpha^* = 10°$$

For $u = 1.64 \cdot 10^{-2}\ S^{-1}$ we will have

$$V_R = V \cdot 0.6 = 120\ \text{km/h}$$

with

$$\alpha = 70°$$
$$\alpha^* = 20°$$

The comparison shows that the second version is superior to the first. In the following, only the second version will be used in all cases.

The angle of climb in the second version is very steep. Results of the same magnitude can be obtained by making flights in one direction with large angles of climb and small angles of descent. This makes it unnecessary to change course constantly.

The derived relationships are retained except for the signs. For the cruising speed we will have, for example,

$$V_R = V \cdot \frac{\sin(\alpha+\alpha^*)}{\sin\alpha + \sin\alpha^*}$$

If the angles are small, we will have

$$V_R = V \frac{\alpha+\alpha^*}{\alpha+\alpha^*} = V$$

An accurate analysis of this type of flight probably also gives a high cruising/8 speed. The accelerations in this version will be very small. This is a fact that is of no inconsiderable importance on long-distance flights. Because of the complicated calculations, this (probably more suitable) version will not be examined further in this paper.

3. Optimum Air Speed

In the above examples, the air speed and lift-drag ratio were given and the most favorable angle of inclination for the flight path was calculated as a function of the particular goal ($\frac{dE}{dh} \to \max$ or $V_R \to \max$). The angles depend, among other things, on the speed and the lift-drag ratio. We shall now assume that the polars assumed for stationary flight are also valid for the given nonsteady flight movements. We will therefore let $\varepsilon = \varepsilon(V)$.

3.1. Maximum Energy Gain with Altitude

In Section 1, with a fixed ε and V, the most favorable angle of climb was calculated from Equation (8):

$$\sin^3\alpha = \frac{g\cdot\varepsilon}{v\cdot u} \cdot \cos\alpha$$

For the most favorable air speed we obtain from the condition $\frac{d}{dv}(\frac{dE}{dh}) = 0$ the relationship

$$\frac{d\varepsilon}{dv} = \frac{u}{g} \sin \alpha \cos \alpha \tag{16}$$

With (8) and (16), we have two equations for determining α and v. If we eliminate α from (8) and (16), we will have

$$\frac{\varepsilon}{v} \cdot \frac{d\varepsilon}{dv} (1 + \frac{v}{\varepsilon} \cdot \frac{d\varepsilon}{dv})^3 = \frac{u^2}{g^2} \tag{17}$$

This equation is very involved for a numerical treatment. The approximation /9 $\alpha = 45°$ should be introduced. As the results in Section 1 indicate, this assumption is justified. Equation (16) then becomes

$$\frac{d}{dv} = \frac{1}{2} \frac{u}{g} \tag{18}$$

The evaluation is performed as follows:

The value a is calculated from the equation $a = 1/2 \frac{u}{g}$ for the corresponding value of u. A straight line with the slope $\varepsilon/y = av + b$ (b is arbitrary) is extended until it intersects the curve $\varepsilon = \varepsilon$ (V). Equation (18) is satisfied at the intersection. All values of interest can be calculated with the values obtained for ε and V (Figure 5). The speeds thus obtained are somewhat too large, since 1/2 is the maximum of the function $\sin \alpha \cdot \cos \alpha$.

The most favorable speed of 390 km/h is obtained for $U = 3.28 \cdot 10^{-2}\ S^{-1}$. The lift-drag ratio is then 1/13.

The most favorable angle of climb is 37°. Our assumption of $\alpha = 45°$ is approximately fulfilled. The other values are

$$\frac{dE}{dh} = 0.14 \text{ Mg}$$

$$\frac{dE}{dt} = 9 \text{ Mg m/s}$$

This corresponds to a rise of 9 m/s in the thermal.

The gain achieved by the increase in air speed is considerable. The optimum air speed decreases with decreasing wind shear. The most favorable values for $u = 1.64 \cdot 10^{-2}\ S^{-1}$ are:

$$V = 260 \text{ km/h}$$
$$\alpha = 40°$$
$$\frac{dE}{dh} = 0.0027 \text{ Mg}$$
$$\frac{dE}{dt} = 1.3 \text{ Mg m/s}$$

3.2. Maximum Air Speed /10

The air speed is calculated according to Section 2 (15) from

$$V_R = v\ (1 - \frac{g\varepsilon}{v \cdot u})$$

For the most suitable air speed the condition $dV_R/dv = 0$ offers the definitive equation

$$\frac{d\varepsilon}{dv} = \frac{u}{g} \tag{19}$$

This equation differs from (18) only by the factor 2. The evaluation is performed analogously to the precedure in Section 3.1. We can see immediately from Figure 5 that for $u = 1.64 \cdot 10^{-2}\ S^{-1}$ the most favorable speed is 390 km/h, while for $u = 3.28 \cdot 10^{-2}\ S^{-1}$ the pilot should fly at the highest possible speed. In the given range, $\frac{d\varepsilon}{dv}$ always remains smaller than $\frac{u}{g}$.

The speeds calculated with approximation (15) are somewhat higher than the optimum speed in reality.

We obtain the following values:

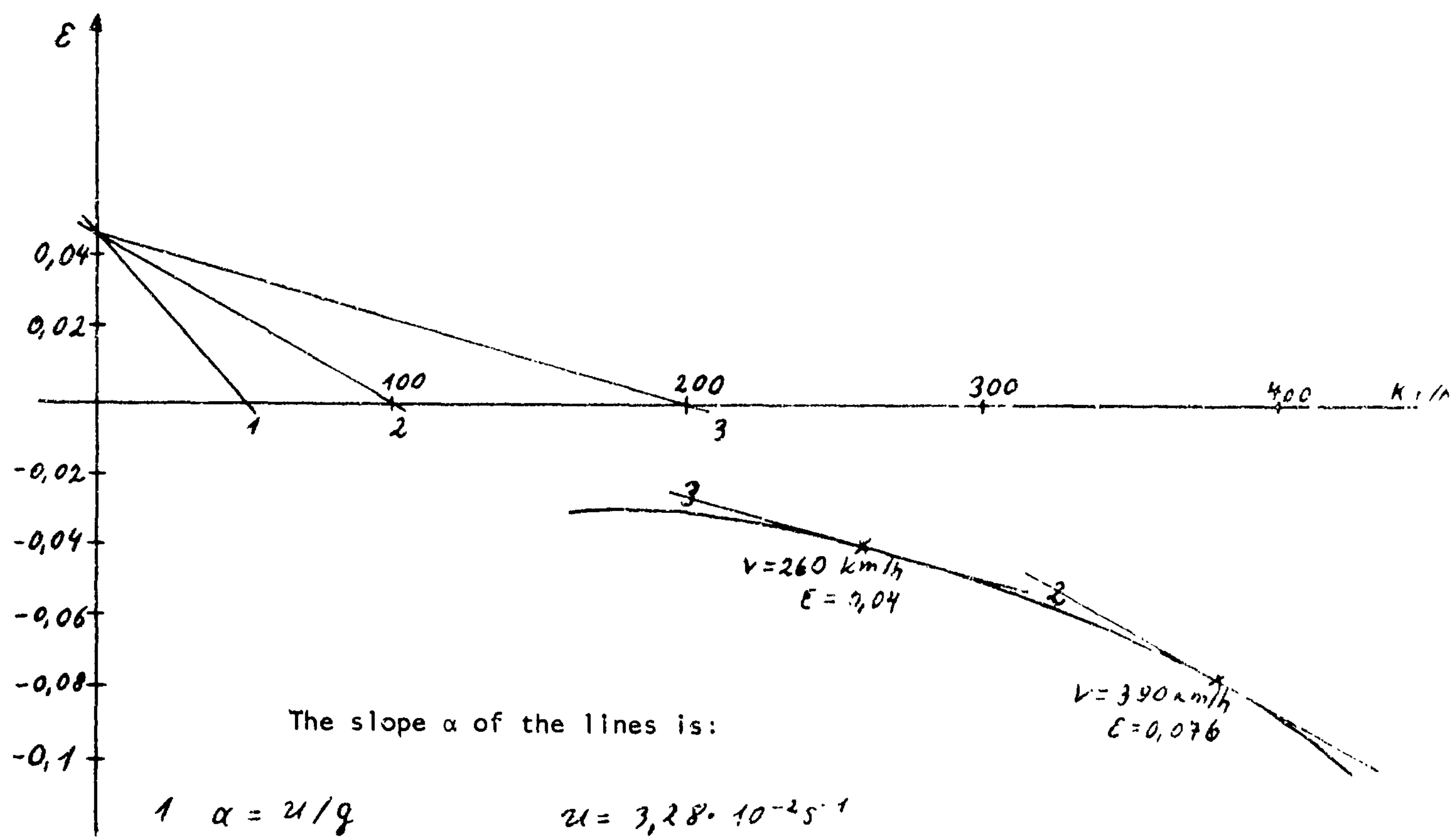
For $u = 3.28 \cdot 10^{-2}\ S^{-1}$, we will have V - 440 km/h.

The air speed is V_R = 320 km/h, and as the angles of inclination we will have

$$\alpha = 75° \qquad \alpha^* = 15°$$

According to (19), for $u = 1.64 \cdot 10^{-2}\ S^{-1}$ we will have an air speed of 390 km/h (V_R = 115 km/h: $\alpha = 57°$, $\alpha^* = 33°$).

Due to the poor approximation, this value is already somewhat further removed from the optimum value. For example, for V = 300 km/h the air speed is already 145 km/h.



The slope α of the lines is:

1 $\alpha = u/g$ $\quad u = 3{,}28 \cdot 10^{-2} s^{-1}$

2 $\alpha = u/g$ $\quad u = 1{,}64 \cdot 10^{-2} s^{-1}$

$\alpha = u/2g$ $\quad u = 3{,}28 \cdot 10^{-2} s^{-1}$

3 $\alpha = u/2g$ $\quad u = 1{,}64 \cdot 10^{-2} s^{-1}$

Figure 5.

4. Flight in the Jet Stream /11

The above estimates indicate that it must be technically possible to make long soaring flights in the region of strong wind shear in the jet stream. To be sure, the flight will differ from the conventional type. There are many more parameters for the pilot to consider. In flights in thermals and rising air currents, the rate-of-climb indicator or "Tevar" shows us how we are to act. In the jet stream, the strong turbulence and the stationary ascending and descending air currents enter as important factors into the effect of wind shear shear studied in this paper. The pilot in any case should have available a device that makes it possible for him to separate the effects of ascending air currents and wind shear. It would be a big help if the value u could be measured. This might be done through (5). We write this equation in the form

$$\frac{dv}{dt} + g \sin \alpha = vu \cos \alpha \sin \alpha - g\varepsilon$$

where $\frac{dv}{dt}$ and $g \cdot \sin \alpha$ have different signs.

The magnitude $\frac{dv}{dt}$ can technically be measured by differentiation of the speed data according to the principle of the rate-of-climb indicator. Then $g \sin \alpha$ is the acceleration in the direction of the flight path. It can be determined when the lifting force is measured for a mass that can move only in the direction of the longitudinal axis of the aircraft.

The difference between the two values is a measure of the additional acceleration that appears as the result of wind shear, less the retardation caused by air resistance. This difference is independent of the existing stationary rising and descending air currents. Usable results, as expected, can be obtained only in the severe retardation and acceleration phases. If the angle of inclination of the flight path is on the order of 45°, we can disregard g in this phase and draw conclusions regarding the value of the wind shear.

For the pilot, the indication /12

$$D = \frac{dv}{dt} + g \sin \alpha$$

would have the following significance:

1. During flight along an inclinced path, D as well as $\frac{dE}{dh}$ and $\frac{dE}{dt}$ are positive.

This is an indication that the energy gain is a consequence of the existing wind shear. A flight maneuver in accordance with the versions described above should be carried out. If D and $\frac{dE}{dh}$ and $\frac{dE}{dt}$ are negative, the direction of flight should be reversed.

2. D is zero, $\frac{dE}{dh}$ and $\frac{dE}{dt}$ are positive. In this case, the energy gain can be attributed to a stationary rise. The pilot should fly in circles or straight ahead. The pilot's stomach will be responsible for this decision.

3. D is positive and $\frac{dE}{dt}$ is less than zero; the energy gain due to wind shear is negated by powerful downward air currents. The pilot must repeat the flight maneuver with alternating climbing and descent.

It should also be pointed out here that flights must be made on sharply inclined paths to determine the wind shear.

A long-distance flight in the jet stream could proceed as follows:

The launching takes place in the lee wave. At certain altitudes proposed by meteorologists, flight along a path inclined at 40-45° is assumed and the value of D is observed. If it is markedly different from zero, the pilot can leave the wave and begin the climbing and descent cycle. As long as D is not equal to zero, the pilot is in a zone of strong wind shear. Even if $\frac{dE}{dh}$ or $\frac{dE}{dt}$ is less than zero, there is no need for concern. If D is equal to zero and $\frac{dE}{dh}$ or $\frac{dE}{dt}$ is positive, the pilot should fly in circles or straight ahead. If he /13 stops climbing, he must resume flight on an inclined path in order to find areas with strong wind shear. This continues until the jet stream is lost. Then the pilot continues flying in the thermal at lower altitude. It is to be expected that in view of the high wind velocity in the jet stream (up to 200 km/h) and the high air speed relative to the air (up to 300 km/h according to this estimate), very high speeds relative to the ground (up to 500 km/h) can be attained. The numerical values obtained constitute a rough approximation. They do show, however, that more attention should be given to dynamic soaring.

All of the relationships obtained indicate that rather fast aircraft may be used for flying in the jet stream. For this reason, jet trainers are very valuable for test flights in the jet stream. The poorer lift-drag ratio that they have relative to gliders is compensated by their higher speed. They can reach the required altitudes without difficulty and can also fly the somewhat complex patterns.

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The gain achieved by the increase in air speed is considerable. The optimum air speed decreases with decreasing wind shear. The most favorable values for $u = 1.64 \cdot 10^{-2}$ S^{-1} are: